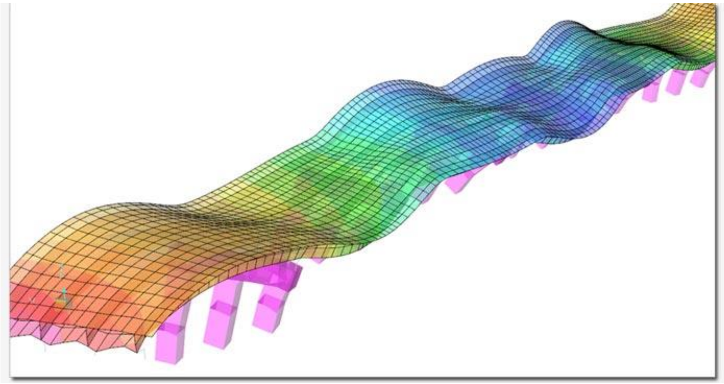
ANALYSIS OF BENT SECTIONS IN SERVICEABILITY LIMIT STATE



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ANALYSIS OF BENT SECTIONS IN SERVICEABILITY LIMIT STATE

Stress analysis in SLS is necessary for:

- **stress limitation**
- **crack control**
- **deflection control**

1. CRACKING BENDING MOMENT

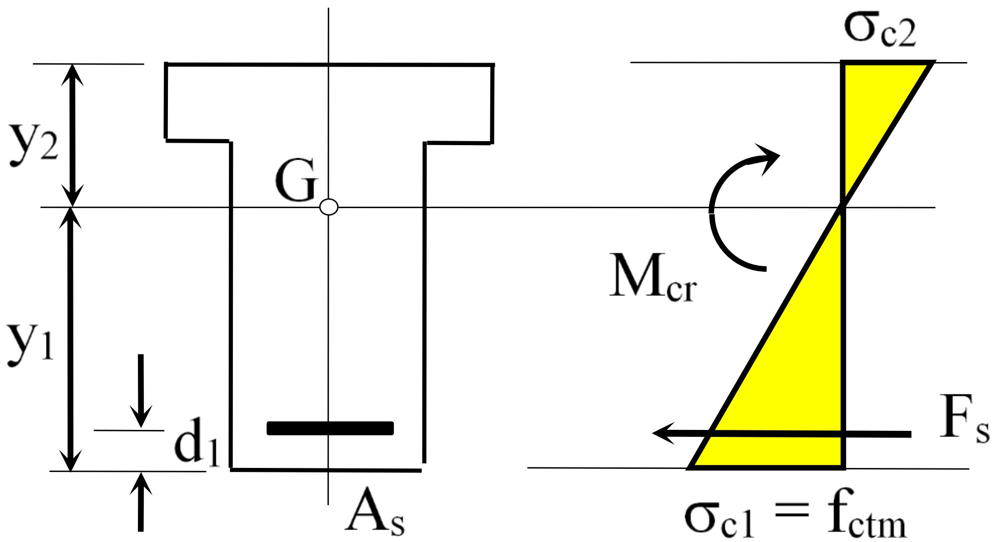
2. STRESSES IN CRACKED SECTION

3. STRESSES LIMITATION

CRACKING BENDING MOMENT

Calculation of cracking bending moment in current sections is based on the hypothesis of **elastic behaviour of concrete**, not considering the plastic deformations of tensioned concrete just before cracking

→ influence of reinforcement on the position of neutral axis can be neglected as well as its contribution to the value of the cracking bending moment



In conformity to Navier relation:

$$M_{cr} = f_{ctm} W_1$$

(Prof. Clipii)

1. CRACKING BENDING MOMENT

2. STRESSES IN CRACKED SECTION

3. STRESSES LIMITATION

STRESSES IN SERVICE STAGE (CRACKED)

SERVICE STAGE ↔ 2nd Stage ↔ cross section is cracked

When determining stresses in concrete and reinforcement the assumptions considered are:

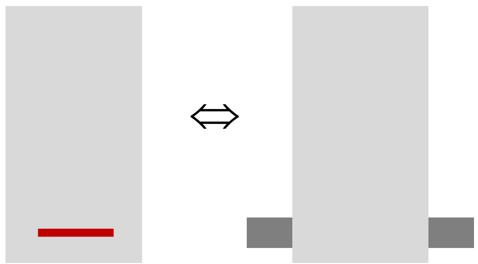
- Plane sections before deformation remain plane after deformation;
- Elastic behaviour for both reinforcement and compressed concrete (Hook's law);
- Neglecting tensioned concrete contribution between cracks;
- the strain in bonded reinforcement, whether in tension or in compression, is the same as that in the surrounding concrete;.

2. Stresses in cracked section / Eforturilor unitare în stadiul de serviciu

At the level of reinforcement, concrete and reinforcement have the same strain:

$\varepsilon_s = \varepsilon_c \quad \rightarrow \quad \sigma_s / E_s = \sigma_c / E_c \quad \rightarrow \quad \sigma_s = \sigma_c (E_s / E_c)$

$\alpha_e = E_s / E_c$ coefficient of equivalence



$\rightarrow F_s = A_s \sigma_s = A_s(\alpha_e \sigma_c) = (\alpha_e A_s)\sigma_c = A_{c,eq}\sigma_c$

$A_{c,eq} = \alpha_e A_s$ → allows replacement of reinforcement area with an equivalent concrete

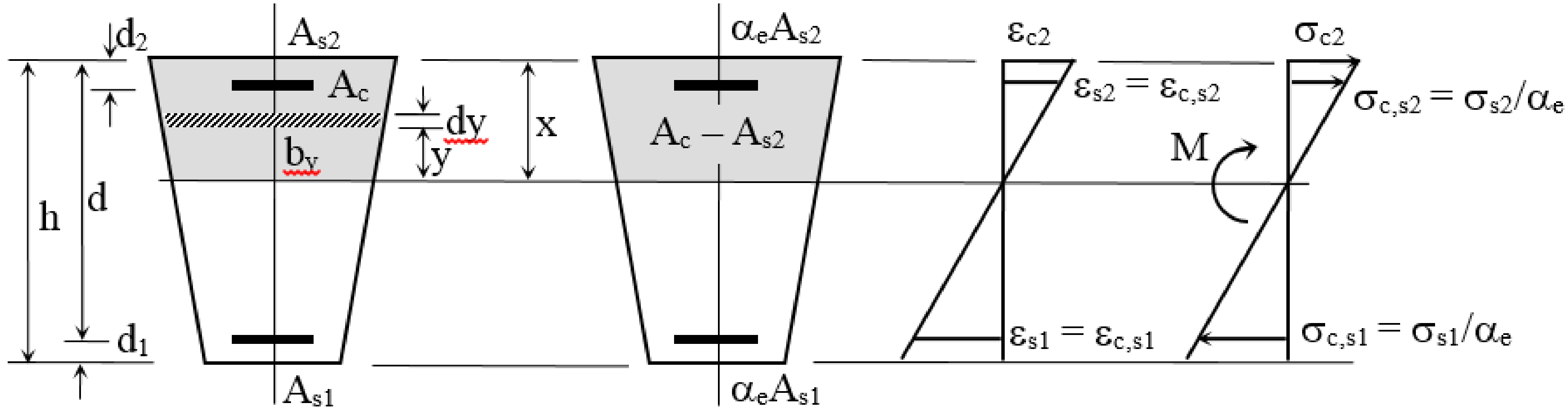
→ to **compute stresses in service stage** (stage II), the **non-homogeneous reinforced concrete section** is replaced by a **homogeneous section of concrete**

2. Stresses in cracked section / Eforturilor unitare în stadiul de serviciu

In servicebility limit stage for stress control, for crack control and deflection control the effect of creep must be considered

$$E_{c,eff} = \frac{E_{cm}}{1 + \varphi(\infty, t_0)}$$

2. Stresses in cracked section / Eforturilor unitare în stadiul de serviciu



Position of neutral axis can be obtained from the equation of first moment of area about neutral axis:

$$\boldsymbol{S_c = S_t}$$

$S_c = S_{cc} - A_{s2}(x - d_2) + \alpha_e A_{s2}(x - d_2)$ first moment of compressed zone

$S_{cc} = \int_0^x y b_y \, d_y$ first moment of compressed concrete area A_c about neutral axis

$S_t = \alpha_e A_{s1}(d - x)$ first moment of tension area

(Prof. Clipii)

2. Stresses in cracked section / Eforturilor unitare în stadiul de serviciu

Compressed zone height x resulting from a 2[nd] order equation:

$$S_{cc} + (\alpha_e - 1)A_{s2}(x - d_2) - \alpha_e A_{s1}(d - x) = 0$$

Stress in **compressed concrete** and **reinforcements**

Navier's formula applied for cracked RC section in bending

$$\sigma_c = \frac{M}{I_{II}} x$$

$$\sigma_{s1} = \alpha_e \sigma_{c,s1} = \alpha_e \frac{M}{I_{II}}(d - x)$$

$$\sigma_{s2} = \alpha_e \sigma_{c,s2} = \alpha_e \frac{M}{I_{II}}(x - d_2)$$

In service stage (stage II) inertia of homogeneous cracked area about neutral axis:

$$I_{II} = I_{cc} + (\alpha_e - 1)A_{s2}(x - d_2)^2 + \alpha_e A_{s1}(d - x)^2$$

where

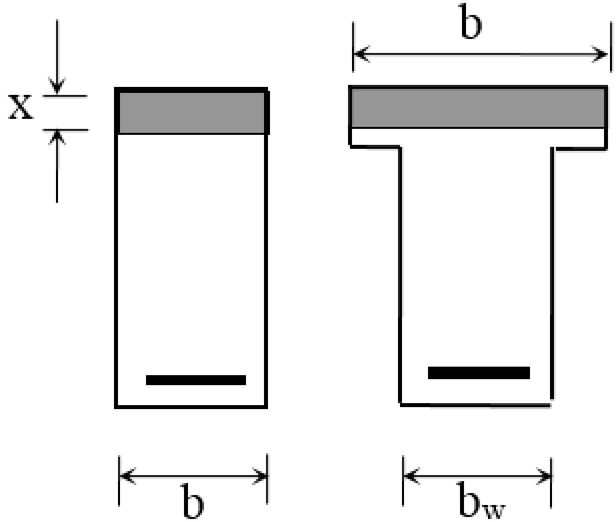
$$I_{cc} = \frac{bx^3}{3} - (b - b_w)\frac{(x - h_f)^3}{3}$$

inertia of compressed concrete area about neutral axis

Inertia of reinforcement area about own axis is negligible. *(Prof. Clipii)*

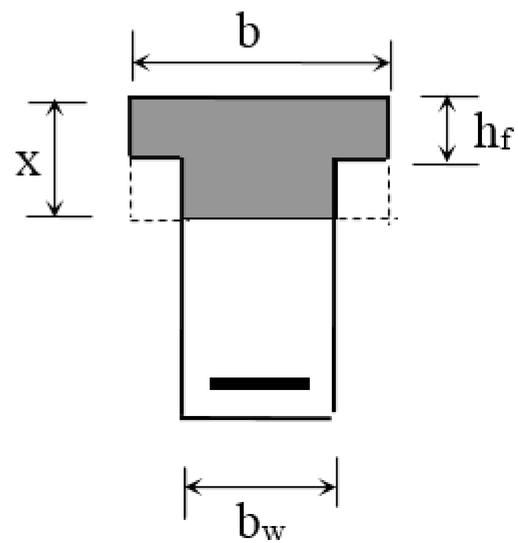
2. Stresses in cracked section / Eforturilor unitare în stadiul de serviciu

For rectangular cross sections and T cross sections



$$S_{cc} = 0{,}5bx^2$$

$$I_{cc} = bx^3/3$$



$$S_{cc} = 0{,}5bx^2 - 0{,}5(b - b_w)(x - h_f)^2$$

$$I_{cc} = bx^3/3 + (b - b_w)(x - h_f)^3/3$$

(Prof. Clipii)

1. CRACKING BENDING MOMENT

2. STRESSES IN CRACKED SECTION

3. STRESSES LIMITATION

3. Stresses limitation / Limitarea eforturilor unitare

Compression stresses in concrete shall be limited to avoid longitudinal cracks or non-linear deformations from creep

1) Under **characteristic combinations** of loads ($G + Q_{k,1} + \psi_{0,i} Q_{k,i}$) for elements exposed to environment of exposure classes XD & XS (chloride attack) and XF (freeze-thaw attack) compression stress is limited to:

$$\sigma_c \leq 0{,}6 f_{ck}$$

Ψ_0 - *ULS, irreversible SLS*
Ψ_1 - *ULS with A and reversible SLS*
Ψ_2 - *ULS with A, reversible SLS, long term effects*

Table A1.1 - Recommended values of ψ factors for buildings

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings, category (see EN 1991-1-1)			
Category A : domestic, residential areas	0,7	0,5	0,3
Category B : office areas	0,7	0,5	0,3
Category C : congregation areas	0,7	0,7	0,6
Category D : shopping areas	0,7	0,7	0,6
Category E : storage areas	1,0	0,9	0,8
Category F : traffic area, vehicle weight ≤ 30kN	0,7	0,7	0,6
Category G : traffic area, 30kN < vehicle weight ≤ 160kN	0,7	0,5	0,3
Category H : roofs	0	0	0
Snow loads on buildings (see EN 1991-1-3)*			
Finland, Iceland, Norway, Sweden	0,70	0,50	0,20
Remainder of CEN Member States, for sites located at altitude H > 1000 m a.s.l.	0,70	0,50	0,20
Remainder of CEN Member States, for sites located at altitude H ≤ 1000 m a.s.l.	0,50	0,20	0
Wind loads on buildings (see EN 1991-1-4)	0,6	0,2	0
Temperature (non-fire) in buildings (see EN 1991-1-5)	0,6	0,5	0

NOTE The ψ values may be set by the National annex.
* For countries not mentioned below, see relevant local conditions.

EN 1990:2002 & CR0-2012

3. Stresses limitation / Limitarea eforturilor unitare

Compression stresses in concrete shall be limited to avoid longitudinal cracks or non-linear deformations from creep

2) Under **quasi-permanent load combination** $(G + \psi_2 Q_k)$ in order to allow the development of a linear creep, the limitation of the compressive stress is:

$$\sigma_c \leq 0{,}45 f_{ck}$$

Ψ_0 - ULS, irreversible SLS
Ψ_1 - ULS with A and reversible SLS
Ψ_2 - ULS with A, reversible SLS, long term effects

Table A1.1 - Recommended values of ψ factors for buildings

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings, category (see EN 1991-1-1)			
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NOTE The ψ values may be set by the National annex.
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EN 1990:2002 & CR0-2012

3. Stresses limitation / Limitarea eforturilor unitare

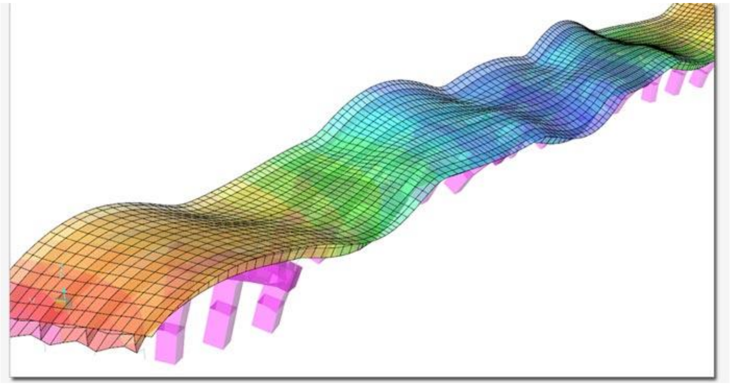
Tensile stress in reinforcement shall be limited in order to avoid inelastic strain, inacceptable crack width or deformations

$\sigma_s \leq 0{,}8 f_{yk}$ - for **characteristic combinations** of loads

$$(G + Q_{k,1} + \psi_{0,i} Q_{k,i})$$

$\sigma_s \leq f_{yk}$ - stresses caused by imposed deformations

THANK YOU FOR YOUR ATTENTION!



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